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Proof $u \sim v \Leftrightarrow u-v \in W$ is an equivalence relation

1. reflexivity: Show that $(u,u) \in \sim$

$u \sim u \Leftrightarrow u-u \in W$

$\Leftrightarrow 0 \in W$ $(u-u=0)$

$\Leftrightarrow T$ (through definition of W as a subspace of V)

2. symmetry: Show that $u \sim v \Leftrightarrow v \sim u$

$u \sim v \Leftrightarrow u-v \in W$ (Def $\sim$)

$\Leftrightarrow -(v-u) \in W$ (commutativity and distributivity)

$\Leftrightarrow v-u \in W$ (Closure under scalar multiplication)

$\Leftrightarrow v \sim u$

3. Transitivity: $a \sim b \wedge b \sim c$

$\Leftrightarrow a-b \in W \wedge b-c \in W$ (Def $\sim$)

$\Leftrightarrow (a-b+b-c) \in W$ (Closure under addition)

$\Leftrightarrow (a-c) \in W$ $(b-b=0)$

$\Leftrightarrow a \sim c$

$\Rightarrow$ $u \sim v \Leftrightarrow u-v \in W$ is an equivalence relation, since it is reflexive, transitive and symmetric