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Pigeonhole Party

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Pick any one mathematician A . There are five other people that either know A or don't know A . So by the pigeonhole principle at least $\lceil 5/2 \rceil = 3$ of them either all know A or all do not know A . Call those three X, Y, Z .

In the first case X, Y, Z all know A . If any pair among them know each other, say X and Y , then A, X, Y are three people who all know each other. If no pair among X, Y, Z know each other, then X, Y, Z are three people who are pairwise strangers. In either case we have a triple that is either all mutual acquaintances or all mutual strangers.

The argument is identical if the three chosen were the ones who do not know A .

Thus among six mathematicians such a triple must exist.